Supersonic Laminar Flow Past a Small Rear-Facing Step

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Theme

A GENERAL unified theory of small steady-state disturbances in compressible boundary layers is applied to the problem of high-speed laminar flow past a small rear-facing step or suction gap. The highly nonuniform flow within the boundary layer and viscous-inviscid interaction are taken into account, as are surface heat and mass transfer, lateral pressure gradient, and upstream influence in the disturbance field. Analytical solutions for the pressure, skin-friction, and heat-transfer perturbations are obtained by Fourier transformation. Comparisons with a variety of experimental data show good agreement.

Content

This paper describes the application of a unified general theory of small disturbances in nonuniform high-speed boundary layers[1] to the case of nonadiabatic laminar flow past a small rearward-facing step or suction gap (Fig. 1). Although this problem has been extensively studied both theoretically and experimentally, these studies have dealt primarily with the Chapman-Korst limit, where the step height h is large compared to the incoming boundary-layer thickness δ and the resulting expansion around the corner is essentially a rotational inviscid flow problem. Much less has been done for high Reynolds number flows in the opposite limit of a small step such that $h/\delta \ll 1$. However, this latter situation is of considerable practical interest; it arises, for example, in connection with possible faults in the structural skin joints on large high-speed vehicles (such as the Space Shuttle) where the heat-transfer and pressure disturbances due to these small steps or gaps is of interest.

We consider the basic compressible boundary-layer flow of a perfect gas in the x-direction past some mean surface ($y = 0$). The Prandtl and Lewis numbers are taken to be unity. Superimposed on this flow are small steady three-dimensional disturbances in velocity (u', v'), pressure (p') and temperature (T') caused by the step, which are governed by a set of linear partial differential equations. Guided by the results of hydrodynamic stability studies, the analysis of these disturbances can be greatly simplified without appreciable loss in accuracy by approximating the mean flow as a rotational plane parallel flow with uniform static pressure p_o and arbitrary lateral variations of density $\rho_o(y)$, velocity $U_o(y)$, Mach number $M_o(y)$, and temperature $T_o(y)$. Furthermore, under the high Reynolds number flow conditions of practical interest, the disturbance field may be resolved into inviscid and viscous components, with the viscous component being significant only in a relatively thin sublayer near the surface wherein the disturbance motion is approximately incompressible with negligible viscous dissipation heating.

These assumptions simplify the linearized compressible Navier-Stokes equations into a set of inviscid disturbance equations including the rotational pressure perturbation relation

$$\frac{\partial^2 p'}{\partial x^2}(1-M_o^2) + \frac{\partial^2 p'}{\partial y^2} - 2\left(\frac{dM_o/dy}{M_o}\right)\frac{\partial p'}{\partial y} = 0 \quad (1)$$

and a set of higher order boundary-layer-type equations governing the viscous heat-conducting disturbance sublayer behavior

$$v_o \frac{\partial^4 v'}{\partial y^4} - \frac{\partial}{\partial x}\left(U_o \frac{\partial^2 v'}{\partial y^2}\right) - v' \frac{d^2 U_o}{dy^2} = 0 \quad (2)$$

$$(\partial u'/\partial x) + (\partial v'/\partial y) = 0 \quad (3)$$

$$v_o \frac{\partial^2 H'}{\partial y^2} \simeq U_o \frac{\partial H'}{\partial x} + v' \frac{dH_o}{dy} \quad (4)$$

with $\partial p'/\partial y \simeq 0$ and $H' \simeq C_p T' + u'U_o$. The solution of Eq. (2) yields the viscous displacement effect and, hence, the effective inner boundary location $y = y^*$ seen by the outer inviscid disturbance flow, which also introduces the effects of upstream influence. The skin-friction and heat-transfer disturbances are then found from the solution of Eqs. (3) and (4). The appropriate wall boundary conditions are no slip $u_w' = 0$ (allowing for a drop in wall location by an amount h at the origin, plus a possible delta function normal perturbation velocity due to suction or blowing) and the thermal boundary conditions for the energy equation that the surface is either at a fixed temperature ($T_w' = 0$) or is insulated $[(\partial T'/\partial y)_w = 0]$.

This boundary value problem is solved by Fourier transformation with respect to x so as to reduce the equations to two sets of *ordinary* differential equations that can be solved both analytically and numerically by well-known methods. Complete details of this analysis are documented in the reference paper. On physical grounds, it is of interest to cite the results for the large scale and small scale wall pressure solutions since these describe, respectively, the behavior far upstream from, and in the immediate vicinity of, the step and analytical solutions can be obtained in both cases. The asymptotic large scale solution is found to be zero for $x > 0$, while upstream of the corner is given by

$$\frac{p_w'}{\gamma P_o} \simeq \frac{-h}{l_u}\left[\frac{M_o^2(y^*) - M_{B_w} M_o(y^*)}{(M_e^2-1)^{1/2}}\right] e^{x/l_u} \quad (5)$$

where l_u is the upstream influence distance and M_{B_w} is the blowing Mach number. This aspect of the solution is thus discontinuous across the step. In the leading approximation for the opposite small scale behavior limit, we get instead that

$$\frac{p'(x, y)}{\gamma P_o} \simeq -\frac{4h}{\pi y_s}[M_o^2(y^*) - M_{B_w} M_o(y^*)]\, e^{kx/y_s} \quad (6)$$

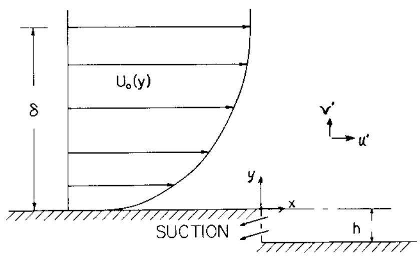

Fig. 1 Flow configuration.

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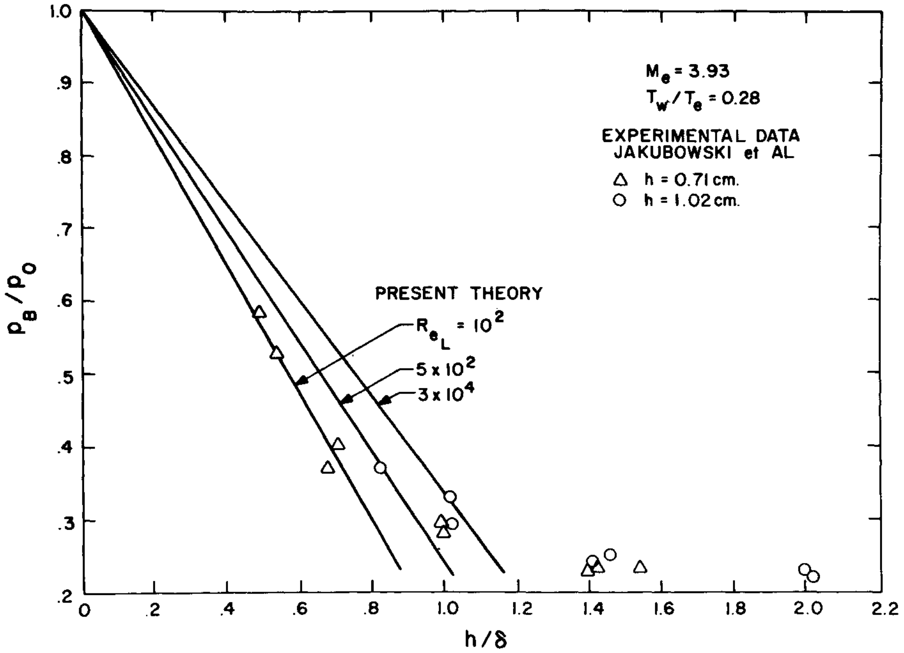

Fig. 2 Base pressure variation vs step height/boundary-layer thickness ratio.

where $k = 1$ and -3 for $x < 0$ and $x > 0$, respectively, and y_s is the sonic height in the Mach number profile. This small scale pressure is continuous past the step (albeit with a discontinuous axial gradient) and acts to locally smooth out the large scale pressure jump.

The variations of base pressure $p_B/p_o = 1-(p_w'/p_o)$ with h/δ, predicted by the present theory for various Reynolds number flows past a cool stepped surface without suction, are given in Fig. 2. Also shown for comparison are some recent experimental data[2] for laminar flows in the range $3 \times 10^2 \leqq Re_L \leqq 3 \times 10^3$. It is seen that the present theory is in good agreement with the data both in trend and magnitude within the region where the small step approximation is expected to be valid. In particular, this data clearly confirms the theoretically-predicted existence of a linear regime for $h/\delta \ll 1$ and indeed shows that the applicability of the linearized approach extends to fairly large values of h/δ ($\approx$0.5). The theoretical base heat-transfer solution $q_B = q_o - q_w'$ is plotted vs h/δ in Fig. 3, along with recent experimental measurements. Although the general trend toward linear behavior at small h/δ is clearly confirmed, the present theory tends to overpredict the heat transfer noticeably more than it does base pressure. This is to be expected since the presence of a recirculation zone downstream of the step (which is not accounted for in the present theory) would cause a significant reduction in base heating for the cited range of step heights.

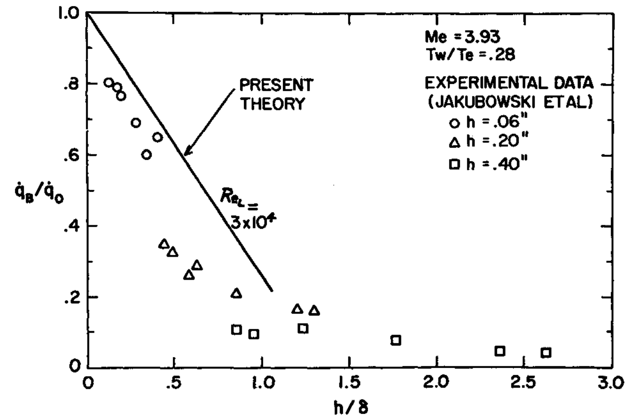

Fig. 3 Base heat transfer vs step height/boundary-layer thickness ratio.

The generally good agreement with the available relevant data suggests that the present theoretical treatment can be profitably extended in several ways to broaden its parametric and physical range of application. First, the presence of a pressure gradient in the basic flow can be readily taken into account, as can a turbulent instead of a laminar boundary-layer profile, since these effects are readily allowed for in the general perturbation equations.[1] Second, to account for nonlinear phenomena associated with finite steps (including possible flow reversal near the surface downstream of the corner) second-order effects can be added in a systematic way. Indeed, it can be shown that the present linear theory is actually an outer solution within which there should be imbedded a small nonlinear inner region centered around the step. Third, it would be of interest to extend the theory to higher Mach numbers by relaxing the non-hypersonic approximations in the manner that has been done in hypersonic boundary-layer stability theory.

References

[1] Inger, G. R., "Three-Dimensional Heat-Transfer and Ablating Disturbances in High Speed Flows," *AIAA Journal*, Vol. 10, No. 12, Dec. 1972, pp. 1641–1646.

[2] Jakubowski, A. K. and Lewis, C. H., "Rearward-Facing Step in Laminar Supersonic Flows with and without Suction," AIAA Paper 73-667, Palm Springs, Calif., 1973.